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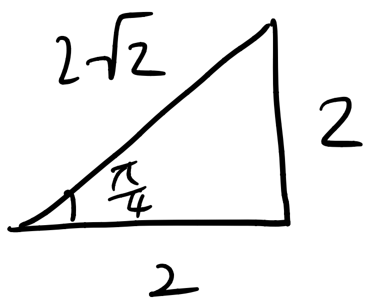
Problem 1

$$f(t) = A\cos(\omega t - \phi)$$

a) $2\cos(3t) + 2\sin(3t)$

$$A = \sqrt{2^2 + 2^2} = 2\sqrt{2} \qquad \phi = \tan^{-1}\frac{2}{2} = \frac{\pi}{4}$$



$$A\cos(\omega t - \phi) = \mathrm{Re}\left(Ae^{i(\omega t - \phi)}\right)$$
$$= \mathrm{Re}\left(e^{i\omega t} \cdot Ae^{-i\phi}\right)$$
$$= \mathrm{Re}\left((\cos(\omega t) + i\sin(\omega t)) \cdot (a - ib)\right)$$

$$\therefore \; f(t) = 2\sqrt{2}\cos\left(3t - \frac{\pi}{4}\right)$$

b) $\sqrt{3}\cos(\pi t) - \sin(\pi t)$

$A = \sqrt{\sqrt{3}^2 + (-1)^2}$

$= 2$

$\phi = \tan^{-1} \frac{-1}{\sqrt{3}}$

$= 2\pi - \frac{\pi}{6}$

$= \frac{11\pi}{6}$

$\therefore 2\cos\left(\pi t - \frac{11\pi}{6}\right)$

c) $\cos\left(t - \frac{\pi}{8}\right) + \sin\left(t - \frac{\pi}{8}\right)$

$= \cos t \cos\frac{\pi}{8} + \sin t \sin\frac{\pi}{8} + \sin t \cos\frac{\pi}{8}$

$- \cos t \sin\frac{\pi}{8}$

$= \cos\frac{\pi}{8}(\cos t + \sin t) + \sin\frac{\pi}{8}(\sin t - \cos t)$

$\therefore$

$= \frac{\pi}{4}$

2

Problem 2

$$\sin x = \operatorname{Im}(e^{ix})$$

$$\int e^{2x} \sin x \, dx$$

$$= \int \operatorname{Im}(e^{(2+i)x}) \, dx$$

$$e^{(2+i)x} = e^{2x}(\cos x + i\sin x) = e^{2x}\cos x + ie^{2x}\sin x$$

$$= \frac{e^{(2+i)x}}{2+i}$$

$$= \frac{(2-i)e^{(2+i)x}}{4-i^2}$$

$$= \frac{(2-i)\,e^{2x}\cdot e^{ix}}{5}$$

$$2e^{ix} = 2(\cos x + i\sin x)$$

$$= \frac{e^{2x}}{5}\cdot \left(2e^{ix} - ie^{ix}\right)$$

$$ie^{ix} = i(\cos x + i\sin x)$$

$$\Rightarrow \operatorname{Im}(e^{(2+i)x}) = \frac{e^{2x}}{5}(i2\sin x - i\cos x)$$

$$= \frac{e^{2x}}{5}(2\sin x - \cos x)$$

$$= \frac{2}{5}e^{2x}\sin x - \frac{1}{5}e^{2x}\cos x$$